九十五學年第二學期 PHYS2320 電磁學 期中考 II 試題(共兩頁)

[Griffiths Ch. 9]　2007/05/17,　10:10am–12:00am, 教師：張存續

記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

◇ Boundary conditions: $D_1^{\perp} - D_2^{\perp} = \sigma_f$, $\mathbf{E}_1^{//} - \mathbf{E}_2^{//} = 0$, $B_1^{\perp} - B_2^{\perp} = 0$, $\mathbf{H}_1^{//} - \mathbf{H}_2^{//} = (\mathbf{K}_f \times \hat{\mathbf{n}})$.

◇ The wave equation: $\nabla^2 f = \dfrac{1}{v^2}\dfrac{\partial^2 f}{\partial t^2}$

1. (12%, 8%)

(a) Which functions satisfy the wave equations? Check by explicit differentiation.

$$f_1(z,t) = Ae^{-b(z-vt)^2}, \quad f_2(z,t) = \frac{A}{b(z-vt)^2+1},$$
$$f_3(z,t) = Ae^{-b(z^2+vt)}, \quad f_4(z,t) = A\sin(bz)\cos^2(bvt).$$

(b) Show that $f_5(z,t) = A\cos(bz)\cos(bvt)$ satisfies the wave equation, and express it as the sum of two traveling waves.

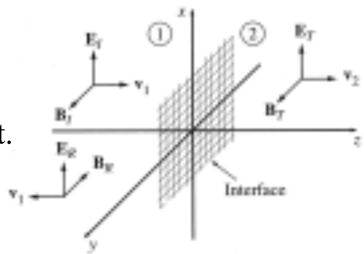
2. (7%, 7%, 6%) The intensity ($I = \langle S \rangle$) of sunlight hitting the earth is about 1320 W/m^2.

(a) Calculate the average energy density $\langle u \rangle$.

(b) Find the amplitude of the electric field E_0.

(c) Find the force on a perfect reflector of area 1 m^2 if sunlight strikes normally on it.

[Hint: $\varepsilon_0 = 8.8\times10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$].



3. (7%, 7%, 6%) A plane wave approaches the interface from the left.

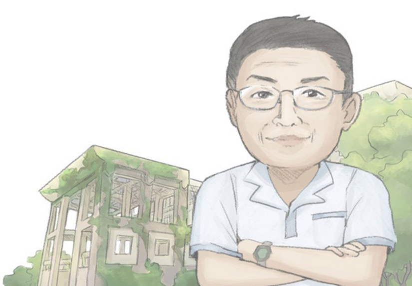
Incident wave: $\begin{cases} \mathbf{E}_I(z,t) = E_{0I}\cos(k_1 z - \omega t)\hat{\mathbf{x}} \\ \mathbf{B}_I(z,t) = \dfrac{1}{v_1}E_{0I}\cos(k_1 z - \omega t)\hat{\mathbf{y}} \end{cases}$

(a) Write down the reflected wave and the transmitted wave in terms of E_{0R} and E_{0T}, respectively.

(b) Write down the four boundary conditions, if there is no free charge and no free current at the interface.

(c) Find the reflection coefficient R and the transmission coefficient T

[Hint: Assume two media are linear.]



4. (12%, 8%) The skin depth is defined as $d \equiv \frac{1}{\kappa} = \frac{1}{\omega}\sqrt{\frac{2}{\varepsilon\mu}}\left[\sqrt{1+(\frac{\sigma}{\varepsilon\omega})^2}-1\right]^{-1/2}$

(a) Show that the skin depth in a poor conductor ($\sigma \ll \omega\varepsilon$) is $(2/\sigma)\sqrt{\varepsilon/\mu}$ and in a good conductor $\sigma \gg \omega\varepsilon$ is $\lambda/2\pi$.

(b) Find the skin depth of salt water ($\varepsilon = 80\varepsilon_0,\ \mu=\mu_0,\ \sigma = 22\ (\Omega\cdot\text{m})^{-1}$) at 60 Hz and 60 GHz. Is it a poor conductor or a good conductor?

[Hint: $\mu_0 = 4\pi\times10^{-7}$ N/A^2].

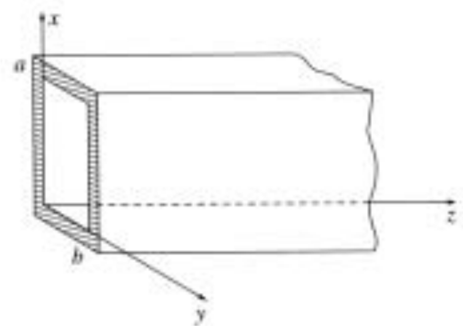
5. (10%, 10%) The field of the TE$_{10}$ waveguide mode is

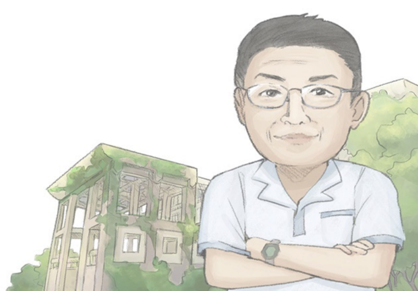
$$B_z = B_0\cos(\pi x/a)e^{i(kz-\omega t)} \text{ and } B_x = \frac{-ika}{\pi}B_0\sin(\pi x/a)e^{i(kz-\omega t)}$$

$$E_y = \frac{i\omega\mu a}{\pi}B_0\sin(\pi x/a)e^{i(kz-\omega t)}$$

(a) If $a = 7.2$ cm and $b = 3.4$ cm , find the cutoff frequency ($f_{mn} = \omega_{mn}/2\pi$) of this mode.

(b) Consider the resonant cavity by closing off the two ends of the waveguide, at z = 0 and z = d, making a perfect conducting empty box. Using the new boundary condition, determine the electric and magnetic fields.





1.

(a)

$f_1(z,t)=Ae^{-b(z-vt)^2}\ \ldots(\bigcirc)$

Let $x=z-vt$, $\frac{\partial f}{\partial z}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial z}=\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial z^2}=\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 x}{\partial z^2}=\frac{\partial^2 f}{\partial x^2}$,

$\frac{\partial f}{\partial t}=\frac{\partial f}{\partial x}\frac{\partial x}{\partial t}=-v\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial z^2}=-v\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 x}{\partial t^2}=v^2\frac{\partial^2 f}{\partial x^2}$

$f_2(z,t)=\frac{A}{b(z-vt)^2+1},\ldots(\bigcirc)$

Let $x=z-vt$，同上 $\therefore \frac{\partial^2 f}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$f_3(z,t)=Ae^{-b(z^2+vt)},\ldots(\mathrm{X})$

$\frac{\partial f}{\partial z}=-2Abze^{-b(z^2+vt)}$, $\frac{\partial^2 f}{\partial z^2}=4Ab^2z^2e^{-b(z^2+vt)}-2Abe^{-b(z^2+vt)}$

$\frac{\partial f}{\partial t}=-Abve^{-b(z^2+vt)},\frac{\partial^2 f}{\partial t^2}=Ab^2v^2e^{-b(z^2+vt)},\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}=Ab^2e^{-b(z^2+vt)}$

$f_4(z,t)=A\sin(bz)\cos^2(bvt).\ldots(\mathrm{X})$

$\frac{\partial^2 f}{\partial z^2}=-b^2A\sin(bz)\cos^2(bvt),$

$\frac{\partial f}{\partial t}=-2bAv\sin(bz)\cos(bvt)\sin(bvt),\frac{\partial^2 f}{\partial t^2}=2b^2Av^2\sin(bz)(\sin^2(bvt)-\cos^2(bvt))$

$\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}=2b^2A\sin(bz)(\sin^2(bvt)-\cos^2(bvt))$

(b)

$\frac{\partial^2 f}{\partial z^2}=-Ab^2\cos(bz)\cos(bvt),\frac{\partial^2 f}{\partial t^2}=-Ab^2v^2\cos(bz)\cos(bvt),\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}=-Ab^2\cos(bz)\cos(bvt)$

$\frac{\partial^2 f}{\partial z^2}-\frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}=0$

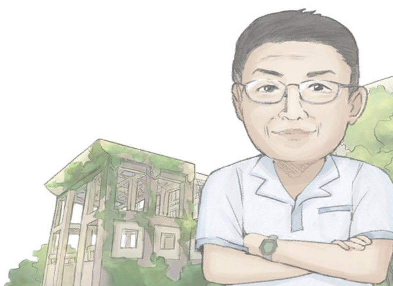
$f=A\cos(bz)\cos(bvt)=\frac{A}{2}(\cos(b(z+vt))+\cos(b(z-vt)))$

2.

(a) $I=\langle S\rangle=c\langle u\rangle$ ➔ $\langle u\rangle=1320/(3\times10^8)=4.4\times10^{-6}$ J/m^3.

(b) $\langle u\rangle=\frac{1}{2}\varepsilon_0E_0{}^2$ ➔ $E_0=\sqrt{(2*4.4*10^{-6})/(8.8*10^{-12})}=1000(\mathrm{V/m})$

(c) $\frac{F}{A}=\frac{\Delta p}{A\Delta t}=\frac{2\Delta U}{Ac\Delta t}=\frac{2SA}{Ac}=\frac{2S}{c}=2u,\ F=8.8\times10^{-6}$ N



3.

(a) Reflected wave: $\begin{cases} \mathbf{E}_R(z,t) = E_{0R}\cos(-k_1 z - \omega t)\hat{\mathbf{x}} \\ \mathbf{B}_R(z,t) = -\dfrac{1}{v_1}E_{0R}\cos(-k_1 z - \omega t)\hat{\mathbf{y}} \end{cases}$

Transmitted wave: $\begin{cases} \mathbf{E}_T(z,t) = E_{0T}\cos(k_2 z - \omega t)\hat{\mathbf{x}} \\ \mathbf{B}_T(z,t) = \dfrac{1}{v_2}E_{0T}\cos(k_2 z - \omega t)\hat{\mathbf{y}} \end{cases}$

(b) For linear media, $\mathbf{D} = \varepsilon\mathbf{E}$ and $\mathbf{B} = \mu\mathbf{H}$

$$\begin{matrix} D_1^{\perp} - D_2^{\perp} = 0 & \mathbf{E}_1^{/\!/} - \mathbf{E}_2^{/\!/} = 0 \\ B_1^{\perp} - B_2^{\perp} = 0 & \mathbf{H}_1^{/\!/} - \mathbf{H}_2^{/\!/} = 0 \end{matrix} \;\Rightarrow\; \begin{matrix} \varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} = 0 & \mathbf{E}_1^{/\!/} - \mathbf{E}_2^{/\!/} = 0 \\ B_1^{\perp} - B_2^{\perp} = 0 & \dfrac{1}{\mu_1}\mathbf{B}_1^{/\!/} - \dfrac{1}{\mu_2}\mathbf{B}_2^{/\!/} = 0 \end{matrix}$$

(c) Normal incident: no components perpendicular to the surface.

Normal boundary conditions: no normal components, so omit.

Tangential boundary conditions:

$$\begin{matrix} \mathbf{E}_1^{/\!/} - \mathbf{E}_2^{/\!/} = 0 \\ \dfrac{1}{\mu_1}\mathbf{B}_1^{/\!/} - \dfrac{1}{\mu_2}\mathbf{B}_2^{/\!/} = 0 \end{matrix} \;\Rightarrow\; \begin{matrix} E_{0I} + E_{0R} = E_{0T} \\ \dfrac{1}{\mu_1 v_1}(\tilde{E}_{0I} - \tilde{E}_{0R}) = \dfrac{1}{\mu_2 v_2}\tilde{E}_{0T} \Rightarrow (\tilde{E}_{0I} - \tilde{E}_{0R}) = \beta\tilde{E}_{0T}, \text{ where } \beta = \dfrac{\mu_1 v_1}{\mu_2 v_2} \end{matrix}$$

$\tilde{E}_{0R} = (\dfrac{1-\beta}{1+\beta})\tilde{E}_{0I}$, Reflection coefficient $R \equiv \dfrac{I_R}{I_I} = (\dfrac{1-\beta}{1+\beta})^2$

$\tilde{E}_{0T} = (\dfrac{2}{1+\beta})\tilde{E}_{0I}$, Transmission coefficient $T \equiv \dfrac{I_T}{I_I} = \dfrac{\varepsilon_2 v_2}{\varepsilon_1 v_1}(\dfrac{2}{1+\beta})^2$

4.

(a) $d \equiv \dfrac{1}{\kappa} = \dfrac{1}{\omega}\sqrt{\dfrac{2}{\varepsilon\mu}}\left[\sqrt{1+(\dfrac{\sigma}{\varepsilon\omega})^2} - 1\right]^{-1/2}$

poor conductor:

$\because \dfrac{\sigma}{\varepsilon\omega} \ll 1, \;\therefore \sqrt{1+(\dfrac{\sigma}{\varepsilon\omega})^2} = 1 + \dfrac{1}{2}(\dfrac{\sigma}{\varepsilon\omega})^2,$

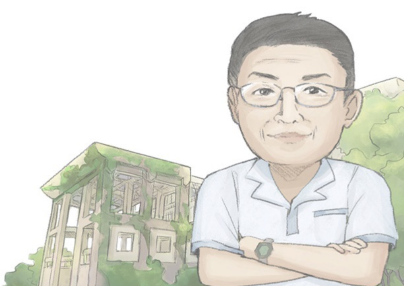
$\therefore\; d \approx \dfrac{1}{\omega}\sqrt{\dfrac{2}{\varepsilon\mu}}\left[1 + \dfrac{1}{2}(\dfrac{\sigma}{\varepsilon\omega})^2 - 1\right]^{-1/2} = \dfrac{1}{\omega}\sqrt{\dfrac{2}{\varepsilon\mu}}\sqrt{2}(\dfrac{\varepsilon\omega}{\sigma}) = \dfrac{2}{\sigma}\sqrt{\dfrac{\varepsilon}{\mu}}$

good conductor:

$\because \dfrac{\sigma}{\varepsilon\omega} \gg 1, \;\therefore d \approx \dfrac{1}{\omega}\sqrt{\dfrac{2}{\varepsilon\mu}}(\dfrac{\sigma}{\varepsilon\omega})^{\frac{-1}{2}} = \sqrt{\dfrac{2}{\omega\mu\sigma}}, \;\because k \simeq \kappa \;,\therefore\; \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{\kappa}, d = \dfrac{1}{k} = \dfrac{\lambda}{2\pi}$

(b)

60Hz: $\dfrac{\sigma}{\varepsilon\omega} = \dfrac{22}{60\cdot 80\cdot\varepsilon_0} = \dfrac{22}{60\cdot 80\cdot 8.85\cdot 10^{-12}} = 5.2*10^8 \gg 1$ …good conductor



$\rightarrow d \approx \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi \cdot 60 \cdot 4\pi \cdot 10^{-7} \cdot 22}} = 13.9(m)$

60GHz: $\frac{\sigma}{\varepsilon\omega} = 5.2 \cdot 10^{8} \cdot \frac{1}{10^{9}} = 0.52 \ldots$poor conductor

$\rightarrow d = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}} = \frac{2}{22}\sqrt{\frac{80 \cdot 8.85 \cdot 10^{-12}}{4\pi \cdot 10^{-7}}} = 2.2 * 10^{-3}(m)$

5.

(a) $\omega_c = \frac{\pi c}{a}, f_c = \frac{c}{2a} = \frac{3*10^{8}}{2*0.072} = 2.08(GHz)$

(b)

$$B_z = B_0^+ \cos(\pi x/a) e^{i(kz-\omega t)} + B_0^- \cos(\pi x/a) e^{i(-kz-\omega t)} = \cos(\pi x/a) e^{-i\omega t}(B_0^+ e^{ikz} + B_0^- e^{-ikz})$$

Boundary:

z=0, $B_z = 0$, $\Rightarrow B_0^+ + B_0^- = 0, \Rightarrow B_0^+ = -B_0^-, \Rightarrow B_z = 2iB_0^+ \cos(\pi x/a) e^{-i\omega t}\sin(kz)$

z=d, $B_z = 0$, $\Rightarrow \sin(kd) = 0, \Rightarrow kd = p\pi, p = 1,2,3\ldots$

$$\Rightarrow B_z = 2iB_0^+ \cos(\pi x/a)\sin(p\pi z/d) e^{-i\omega t}$$

$$\begin{aligned} B_x &= \frac{-ika}{\pi} B_0^+ \sin(\pi x/a) e^{i(kz-\omega t)} + \frac{ika}{\pi} B_0^- \sin(\pi x/a) e^{i(-kz-\omega t)} = \frac{-ika}{\pi}\sin(\pi x/a) e^{-i\omega t}(B_0^+ e^{ikz} - B_0^- e^{-ikz}) \\ &= \frac{-ika}{\pi}\sin(\pi x/a) e^{-i\omega t}(B_0^+ e^{ikz} + B_0^+ e^{-ikz}) = \frac{-2ika}{\pi} B_0^+ \sin(\pi x/a)\cos(kz) e^{-i\omega t} \\ &= \frac{-2ia}{\pi}\left(\frac{p\pi}{d}\right) B_0^+ \sin(\pi x/a)\cos(p\pi z/d) e^{-i\omega t} \end{aligned}$$

$$\begin{aligned} E_y &= \frac{i\omega\mu a}{\pi} B_0^+ \sin(\pi x/a) e^{i(kz-\omega t)} + \frac{i\omega\mu a}{\pi} B_0^- \sin(\pi x/a) e^{i(-kz-\omega t)} = \frac{i\omega\mu a}{\pi}\sin(\pi x/a) e^{-i\omega t}(B_0^+ e^{ikz} + B_0^- e^{-ikz}) \\ &= \frac{i\omega\mu a}{\pi}\sin(\pi x/a) e^{-i\omega t}(B_0^+ e^{ikz} - B_0^+ e^{-ikz}) = \frac{-2\omega\mu a}{\pi} B_0^+ \sin(\pi x/a)\sin(kz) e^{-i\omega t} \\ &= \frac{-2\omega\mu a}{\pi} B_0^+ \sin(\pi x/a)\sin(p\pi z/d) e^{-i\omega t} \end{aligned}$$